Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

Improvement in Separation of Concentric-Tube Thermal Diffusion Columns with Viscous Heat Generation under Consideration of the Curvature Effect

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To cite this Article Yeh, Ho-Ming and Tsai, Shau-Wei(1981) 'Improvement in Separation of Concentric-Tube Thermal Diffusion Columns with Viscous Heat Generation under Consideration of the Curvature Effect', Separation Science and Technology, 16: 1, 63 — 73

To link to this Article: DOI: 10.1080/01496398108070225

URL: http://dx.doi.org/10.1080/01496398108070225

SEPARATION SCIENCE AND TECHNOLOGY, 16(1), pp. 63–73, 1981

Improvement in Separation of Concentric-Tube Thermal Diffusion Columns with Viscous Heat Generation under Consideration of the Curvature Effect

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Abstract

The separation efficiency of concentric-tube thermal diffusion columns with viscous heat generation has been investigated under consideration of the curvature effect. The results were obtained graphically and were compared with those obtained by Yeh and Cheng in which they neglected the curvature effect. Considerable improvement in separation was obtained by employing a rotary column instead of a stationary one.

INTRODUCTION

Thermal diffusion is a well-established method for separating isotopes. It was used to separate uranium isotopes at Oak Ridge Laboratory in World War II. For separation of hydrogen isotopes, this method is particularly attractive because of the large ratio in molecular weight (*20*).

The convective currents in thermal diffusion columns have two conflicting effects: the desirable cascading effect and the undesirable remixing effect. It has been found that proper adjustment of velocity profile and temperature profile might effectively improve separation. Several improved columns (*2–6*, *9–14*, *16*) have been developed for this purpose.

Sullivan, Ruppel, and Willingham (*7*) found from their experimental work that the rotary thermal diffusion column gave better separation than the stationary one, especially for highly viscous fluids. Later, the column theory was given by Yeh and Cheng (*13*). In their theoretical work the effect of curvature on the transport coefficients of thermal diffusion in the rotary concentric-tube column was neglected. However, as the ratio of tube diameters is far from unity, the effect of curvature must be taken into consideration to prevent a serious error from occurring. It is the purpose of this work to develop the complete theory and investigate the improvement in the degree of separation of a rotary thermal diffusion column under consideration of the curvature effect.

COLUMN THEORY

Consider a concentric-tube thermal diffusion column with the surfaces of the inner and outer cyclinders maintained at T_1 and T_2 as shown in Fig. 1(b). As the outer cylinder rotates, each cylindrical shell of fluid rubs against an adjacent shell of fluid. This rubbing together of adjacent layers of fluid produces heat. The temperature distribution for an incompressible Newtonian fluid may be solved from the energy balance Eq. (1). The solution is

$$T = T_2 + \Delta T(-N\xi^{-2} + S\ln\xi + N + 1) \tag{1}$$

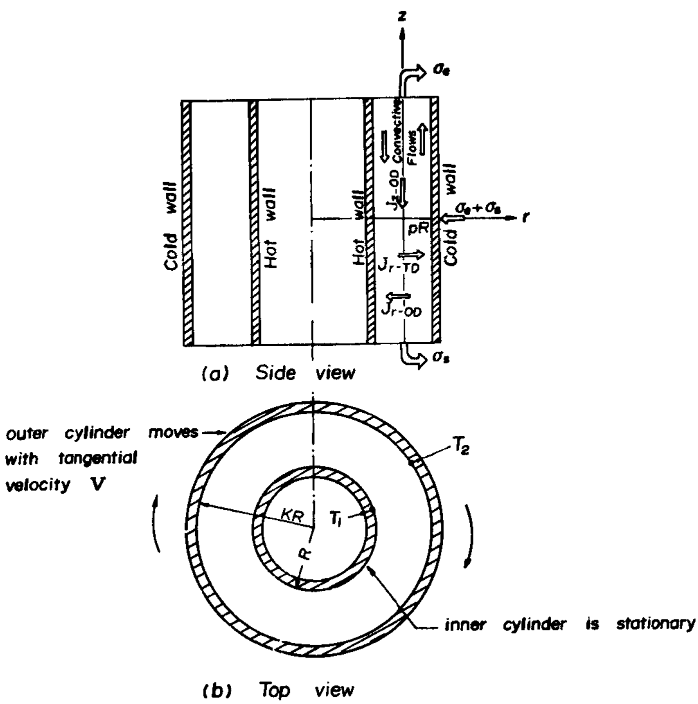

FIG. 1. Schematic diagram of a concentric-tube thermal diffusion column with inner cylinder stationary and outer cylinder rotating with tangential velocity V.

where

$$N = k^2\mathrm{Br}/(k^2 - 1)^2 \tag{2}$$

$$S = \frac{N(1 - k^2) - k^2}{k^2 \ln k} \tag{3}$$

Applying the appropriate equations of motion and energy gives the following steady-state velocity profile. In the enriching section:

$$\begin{aligned} V_z = \frac{\beta_{\bar{T}} g R^2(\Delta T)}{4\mu}\Big\{&(A_1 + S - N - 1)(\xi^2 - 1) - S\xi^2 \ln \xi + 2N(\ln \xi)^2 \\ &- \frac{\ln \xi}{\ln k}[A_1(k^2 - 1) + S(k^2 - k^2 \ln k - 1) \\ &+ N(2(\ln k)^2 - k^2 + 1) - k^2 + 1]\Big\} \end{aligned} \tag{4}$$

where

$$\begin{aligned} A_1 &= \frac{T - T_2}{\Delta T} \\ &= 1 + (\ln k)\Big\{8\mu\sigma_e/\pi R^4 \rho \beta_{\bar{T}} g(\Delta T) + S\Big[\frac{7}{4}k^4 - k^2 - \frac{3}{4} - k^4 \ln k \\ &\quad - \frac{(k^2 - 1)^2}{\ln k}\Big] + N\Big[2(k^2 + 1)\ln k - 2k^2 - k^4 + 3 + \frac{(k^2 - 1)^2}{\ln k}\Big]\Big\} \\ &\quad \Big/ \{(k^2 - 1)[k^2 - 1 - (k^2 + 1)\ln k]\} \end{aligned} \tag{5}$$

The above equations apply to the stripping section with σ_e replaced by $-\sigma_s$. The equation of separation may then be obtained by following the same procedures performed by Yeh (*10, 17*). The solution is

$$\Delta_1 = \frac{H}{2\sigma}\left[1 - \exp\left(\frac{-\sigma L}{2K}\right)\right] \tag{6}$$

in which

$$H = H_0 F(k, \mathrm{Br}) \tag{7}$$

$$K = K_0 G(k, \mathrm{Br}) \tag{8}$$

$$H_0 = \frac{2\pi R \alpha \rho \beta_{\bar{T}} g(\Delta T)^2[R(k - 1)]^3}{6!\mu \bar{T}} \tag{9}$$

$$K_0 = \frac{2\pi R \rho \beta_{\bar{T}}^2 g^2(\Delta T)^2[R(k - 1)]^7}{9! D\mu^2} \tag{10}$$

$$F(k, \mathrm{Br}) = \frac{180}{(k-1)^5}\int_1^k \left[N\left(\frac{1}{p^2} - \frac{1}{\xi^2}\right) + S \ln\frac{\xi}{p}\right]\left[(A_2 + S - N - 1)\right.$$
$$\left. \times (\xi^2 - 1) - S\xi^2 \ln \xi + 2N(\ln \xi)^2 - \frac{A_3}{\ln k}\ln \xi\right]\xi\, d\xi \quad (11)$$

$$G(k, \mathrm{Br}) = \frac{-22680}{(k-1)^7}\int_1^k \left[(A_2 + S - N - 1)(\xi^2 - 1) + 2N(\ln \xi)^2\right.$$
$$\left. - S\xi^2 \ln \xi - \frac{A_3}{\ln k}\ln k\right]\left\{(A_2 + S - N - 1)\left(\frac{\xi^4}{16} - \frac{\xi^2}{4}\right.\right.$$
$$\left. + \frac{1}{4}\ln\frac{\xi}{p} - \frac{p^4}{16} + \frac{p^2}{4}\right) + 2N\left[\frac{\xi^2}{4}(\ln \xi)^2 - \frac{\xi^2}{2}\ln \xi\right.$$
$$\left. + \frac{3}{8}(\xi^2 - p^2) - \frac{1}{4}\ln\frac{\xi}{p} - \frac{1}{4}p^2(\ln p)^2 + \frac{p^2}{2}\ln p\right]$$
$$- S\left[\frac{\xi^4}{16} - \frac{1}{36}(\xi^4 - p^4) + \frac{1}{16}\ln\frac{\xi}{p} - \frac{p^4}{16}\ln p - \left(\frac{A_3}{\ln k}\right)\right]$$
$$\left. \times \left[\frac{\xi^2}{4}\ln \xi - \frac{1}{4}(\xi^2 - p^2) + \frac{1}{4}\ln\frac{\xi}{p} - \frac{p^2}{4}\ln p\right]\right\}\xi\, d\xi \quad (12)$$

$$A_2 = A_1|_{\sigma_e = 0} = A_1|_{\sigma_s = 0} \quad (13)$$

$$A_3 = A_2(k^2 - 1) + S(k^2 - k^2 \ln k - 1) + N[2(\ln k)^2 - k^2 + 1]$$
$$- k^2 + 1 \quad (14)$$

In obtaining the above solution, several assumptions were made:

1. The feed was introduced at the midpoint of the column.
2. The flow rates in both sections were kept the same, i.e.,

$$\sigma_e = \sigma_s = \sigma$$

3. The concentration anywhere in the column was between 0.3 and 0.7 weight fraction.
4. Practically, moderate flow was considered and the flow rate terms in the transport coefficients could be neglected.
5. Diffusion in the transport direction was neglected compared with the convective term.

The integration of the modifying factors $F(k, \mathrm{Br})$ and $G(k, \mathrm{Br})$ is simple in principle but more complicated in execution. For convenience, the integration was performed numerically with the Brinkman number as a parameter. The results were plotted in Figs. 2 and 3. It is shown in both figures that $F(k, \mathrm{Br})$ and $G(k, \mathrm{Br})$ increase as the k value or the

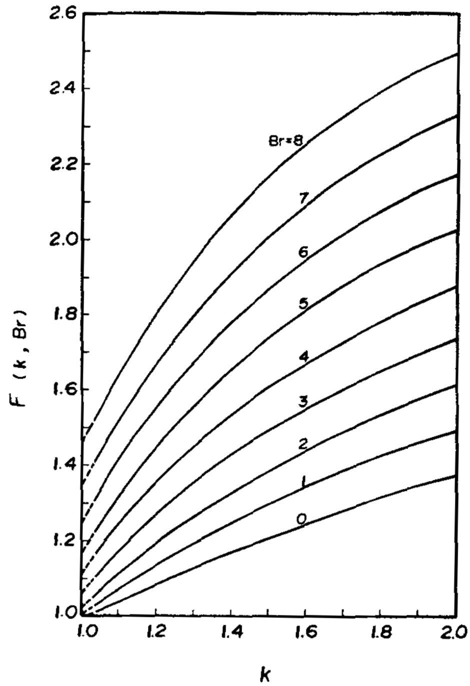

FIG. 2. Graphical representation of modifying factor $F(k, \mathrm{Br})$.

Brinkman number does. When the k value approaches unity, the concentric-tube column is nearly a flat plate. At $k = 1.0$, either the annular spacing or the curvature of the concentric-tube column is zero and the thermal diffusion column is exactly a flat plate. Since the values of the modifying factors cannot be calculated at the singular point $k = 1.0$, the extensions of both values are shown by dotted line in the direction toward $k = 1.0$.

For a nearly flat-plate column, $k \to 1$, the solution was obtained by Yeh and Cheng (*13*) as

$$\Delta_2 = \frac{H_0'\left(1 + \frac{1}{140}\mathrm{Br}^2\right)}{2\sigma\left(1 + \frac{\Delta T}{12T_m}\mathrm{Br}\right)}\left[1 - \exp\left(\frac{-\sigma L}{2K_0\left(1 + \frac{1}{1100}\mathrm{Br}^2\right)}\right)\right] \tag{15}$$

in which

$$H_0' = \frac{2\pi R\alpha\rho\beta_T g(\Delta T)^2[R(k-1)]^3}{6!\mu T_m} \tag{16}$$

The term $(1 - (11/400)\mathrm{Br}^2)$ in the solution of Yeh and Cheng (*13*) is

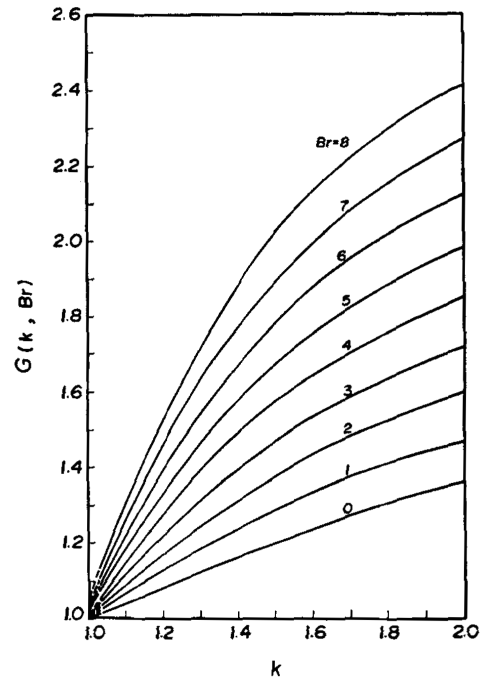

FIG. 3. Graphical representation of modifying factor $G(k, \mathrm{Br})$.

now corrected to $(1 + (1/1100)\mathrm{Br}^2)$ after careful calculation. Besides, during the derivation of equations, Yeh and Cheng (*13*) considered $\bar{T}$ as the average temperature and obtained the relation $\bar{T} = T_m(1 + (\Delta T/12T_m)\mathrm{Br})$. Since the actual reference temperature $\bar{T}$ should be calculated from Eq. (5) once the flow rate is known, the correct form of Eq. (15) must be

$$\Delta_3 = \frac{H_0\left(1 + \frac{1}{140}\mathrm{Br}^2\right)}{2\sigma}\left[1 - \exp\left(\frac{-\sigma L}{2K_0\left(1 + \frac{1}{1100}\mathrm{Br}^2\right)}\right)\right] \tag{17}$$

THE IMPROVEMENT OF SEPARATION

Let us define dimensionless flow rate and reduced seperation by

$$\sigma' = \sigma L/K_0 \tag{18}$$

$$\Delta' = \Delta\sigma/H_0 \tag{19}$$

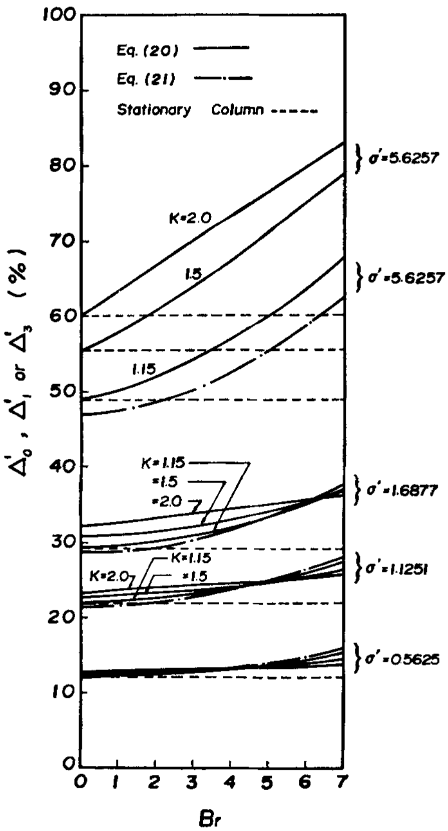

FIG. 4. Effect of Brinkman number on the separation with flow rates as the parameter.

Equations (6) and (17) can be rewritten as

$$\Delta_1' = \frac{F(k, \mathrm{Br})}{2}\left[1 - \exp\left(\frac{-\sigma'}{2G(k, \mathrm{Br})}\right)\right] \tag{20}$$

$$\Delta_2' = \frac{\left(1 + \frac{1}{140}\mathrm{Br}^2\right)}{2}\left\{1 - \exp\left(\frac{-\sigma'}{2\left(1 + \frac{1}{1100}\mathrm{Br}^2\right)}\right)\right\} \tag{21}$$

Figure 4 shows some examples of graphical representations of Eqs. (20) and (21). It is found that the reduced separation increases as the Brinkman number increases at any value of k. When the dimensionless flow rate is greater than unity, the effect of curvature on the degree of separation should be considered. Improvement is also found by employing a rotary column instead of using a stationary one.

The improvement in separation of concentric-tube columns with their outer wall rotated is best illustrated by calculating the percentage increase in separation based on the stationary column:

$$I = \frac{\Delta - \Delta_0}{\Delta_0} \tag{22}$$

where Δ_0 is the degree of separation obtained from the stationary column (*16*) or from Eq. (6) by setting Br = 0.

For the purpose of illustration, let us assign the numerical values for separation of heavy water as follows: $\alpha = 0.0184$, $g = 10^3$ cm/s^2, $\beta_{\bar{T}} = 5 \times 10^{-3}$ g/(cm^3)(°C), $\rho = 1$ g/cm^3, $D = 5 \times 10^{-5}$ cm^2/s, $L = 500$ cm, $\Delta T = 60$°C, $\mu = 1.2$ cP, $R = 0.5$ cm, $\bar{T} = 330$ K, $k = 1.1$, Br = 7. Substituting these values into Eqs. (8) and (10), we obtain

$$H_0 = 2.74 \times 10^{-3} \text{ g/min}$$

$$K_0 = 5.07\text{(g)(cm)/min}$$

From the above values the degree of separation and the improvement of separation can be calculated under different flow rates. The results are presented in Table 1.

CONCLUSION

On the basis of this study, the following conclusions and discussions are reached:

1. The effect of curvature on separation must be taken into consideration once the reduced flow rate is greater than unity.
2. The solution obtained by Yeh and Cheng (*13*) is a special case. The original equation of separation has been corrected.

TABLE 1

Comparison of Separation

$\sigma \times 10^3$ (g/min)	Stationary column, Δ_0 (%)	Rotary column Δ_1 (%)	Rotary column I_1 (%)
1	6.60	8.20	24.24
2	6.43	8.04	25.04
3	6.27	7.88	25.68
4	6.12	7.72	26.14
5	5.99	7.58	26.54

3. During the derivation of equations, the most important assumptions are that the concentration in the column is anywhere between 0.3 and 0.7 weight fraction, and that only moderate flow rates are considered. Separation theories applicable to the whole range of concentration in flat-plate thermal diffusion columns have been developed (*5*, *18*, *19*).
4. Although the degree of separation increases as the Brinkman number increases, there still exist critical values of this number for the laminar flow range. The critical values were obtained experimentally by Schultz-Grunow (*8*).
5. Considerable improvement in separation was obtained by employing a rotary column instead of a stationary one, especially for highly viscous fluids.

SYMBOLS

A_1, A_2, A_3	system constants, evaluated by Eqs. (5), (11), (12)
Br	Brinkman number, defined by $\mu V^2/\lambda(\Delta T)$
D	ordinary diffusion coefficient
$F(k, \mathrm{Br})$	modifying factor, evaluated by Eq. (13)
$G(k, \mathrm{Br})$	modifying factor, evaluated by Eq. (14)
g	gravitational acceleration
H, H_0, H_0'	system constants, evaluated by Eqs. (7), (9), (16)
I	improvement of separation defined by Eq. (22)
$J_{r-\mathrm{OD}}$	mass flux of composition 1 in r-direction due to ordinary diffusion
$J_{r-\mathrm{TD}}$	mass flux of composition 1 in r-direction due to thermal diffusion
$J_{z-\mathrm{OD}}$	mass flux of composition 1 in z-direction due to ordinary diffusion
K, K_0	system constants, evaluated by Eqs. (8), (10)
L	column height
N	dimensionless group defined by Eq. (2)
p	$(k + 1)/2$
R	outside radius of inner tube
kR	inside radius of outer tube
r	axis of radial direction
S	dimensionless group defined by Eq. (3)
T	absolute temperature
T_1, T_2	temperature of hot, cold wall
T_m	$(T_1 + T_2)/2$
ΔT	$T_1 - T_2$

$\bar{T}$	reference temperature defined by Eq. (5)
V	tangential velocity of outer tube rotated
v_z	convective velocity distribution
z	axis of transport direction

Greek Letters

α	thermal diffusion constant
$\beta_{\bar{T}}$	$-(\partial\rho/\partial T)_{\bar{T}}$
Δ	difference of fraction of component 1 between top and bottom products
Δ_0	Δ obtained from stationary column
$\Delta_1, \Delta_2, \Delta_3$	Δ obtained from Eqs. (6), (15), (17)
Δ'	reduced separation defined by Eq. (19)
λ	thermal conductivity of fluid
μ	viscosity of fluid
ξ	r/R
ρ	mass density
σ	mass flow rate
σ_e, σ_s	mass flow rate in the enriching, stripping section
σ'	dimensionless flow rate defined by Eq. (18)

Acknowledgment

The authors wish to express their thanks to The Chinese National Science Council for financial aid.

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Received by editor May 5, 1980